C2 Exercise 7B (simple algebraic approaches to G.Ps)

Note Title 27/04/2013

1 Find the sixth, tenth and n^{th} terms of the following geometric sequences.

a $2, 6, 18, 54, \ldots$

$a = 2 \quad r = 3 \quad$ general rule is $u_n = ar^{n-1}$

sixth term $\quad u_6 = ar^5 = 2 \times 3^5 = 2 \times 243 = 486$

tenth term $\quad u_{10} = ar^9 = 2 \times 3^9 = 2 \times 19683 = 39366$

n^{th} term $\quad u_n = ar^{n-1} = 2 \times 3^{n-1}$

b $100, 50, 25, 12.5, \ldots$

$a = 100 \quad r = \frac{1}{2}$ or 0.5

sixth term $\quad u_6 = ar^5 = 100 \times \left(\frac{1}{2}\right)^5 = 3.125$ or $3\frac{1}{8} = \frac{25}{8}$

tenth term $\quad u_{10} = ar^9 = 100 \times \left(\frac{1}{2}\right)^9 = 0.1953125$ or $\frac{25}{128}$

n^{th} term $\quad u_n = ar^{n-1} = 100 \times \left(\frac{1}{2}\right)^{n-1} = \frac{25}{2^{n-3}}$.

1c $1, -2, 4, -8, \ldots$

$a = 1 \quad r = -2$

sixth term $\quad u_6 = 1 \times (-2)^5 = 1 \times (-32) = -32$

tenth term $\quad u_{10} = 1 \times (-2)^9 = 1 \times (-512) = -512$

n^{th} term $\quad u_n = 1 \times (-2)^{n-1} = (-2)^{n-1}$

1d $1, 1.1, 1.21, 1.331, \ldots$

$a = 1 \quad r = 1.1$

sixth term $\quad u_6 = ar^5 = 1 \times (1.1)^5 = 1.61051$

tenth term $\quad u_{10} = ar^9 = 1 \times (1.1)^9 = 2.357947691$

n^{th} term $\quad u_n = ar^{n-1} = 1 \times (1.1)^{n-1} = \left(\frac{11}{10}\right)^{n-1}$

2 The n^{th} term of a GP is $2\times(5)^n$.
Find the first and 5^{th} terms.

first term $u_1 = 2\times 5^1 = 10$

fifth term $u_5 = 2\times 5^5 = 6250$

3 The sixth term of a GP is 32 and the third term is 4.
Find the first term and the common ratio.

$u_1 = a$
$u_2 = ar$
$u_3 = ar^2 = 4$
$u_4 = ar^3$
$u_5 = ar^4$
$u_6 = ar^5 = 32$
...

$\frac{u_6}{u_3} = \frac{32}{4} = 8$

but $\frac{u_6}{u_3} = \frac{ar^5}{ar^2} = r^3$

so $r^3 = 8$ and $r = \sqrt[3]{8} = 2$

You can now fill in the rest of the table working backwards where necessary to get:

$u_1 = a = 1$
$u_2 = ar = 2$
$u_3 = ar^2 = 4$
$u_4 = ar^3 = 8$
$u_5 = ar^4 = 16$
$u_6 = ar^5 = 32$

So $a = 1$
and $r = 2$

If you're confident with the algebra you can do this much more directly - that's fine - I'm showing more detail for those that find this difficult.

4 Given that the first term of a GP is 4 and the third term is 1. Find possible values for the sixth term.

$u_1 = a = 4$

$u_3 = ar^2 = 1$

$\frac{u_3}{u_1} = \frac{ar^2}{a} \Rightarrow r^2 = \frac{1}{4} \Rightarrow r = \pm\frac{1}{2}$

$u_6 = ar^5 = 4\left(\frac{1}{2}\right)^5$ or $4\left(\frac{-1}{2}\right)^5$

so u_6 could be $\frac{1}{8}$ or $-\frac{1}{8}$

5 The expressions $x-6$, $2x$ and x^2 form the first three terms of a GP. By calculating two different expressions for the common ratio, form and solve an equation in x to find possible values of the first term.

$u_1 = x-6 = a$

$u_2 = 2x = ar$

$u_3 = x^2 = ar^2$

so ① $(x-6)r = 2x$ and ② $(2x)r = x^2$

to get from 1st to 2nd term you multiply by the common ratio.

to get from 2nd to 3rd term you multiply by the common ratio.

rearrange ① to give $r = \frac{2x}{x-6}$

and ② to give $r = \frac{x^2}{2x}$

So $\frac{2x}{x-6} = \frac{x^2}{2x}$

$\Rightarrow 4x^2 = x^2(x-6)$

$\Rightarrow 4 = x-6 \Rightarrow x = 10$ or $x^2 = 0 \Rightarrow x = 0.$

Since $u_1 = x - 6$ the possible values for the first term are 4 and -6.